Maximálně symetrické prostory

Rovina E^2

translace a rotace

$$\mathcal{X}_{x_0} : z \to \mathcal{X}_{x_0} z$$

$$x(\mathcal{X}_{x_0} z) = x(z) + x_0 \qquad y(\mathcal{X}_{x_0} z) = y(z)$$

$$\mathcal{Y}_{y_0} : z \to \mathcal{Y}_{y_0} z$$

$$x(\mathcal{Y}_{y_0} z) = x(z) \qquad y(\mathcal{Y}_{y_0} z) = y(z) + y_0$$

$$\mathcal{R}_{\varphi_0} : z \to \mathcal{R}_{\varphi_0} z$$

$$x(\mathcal{R}_{\varphi_0} z) = \cos\varphi_0 \, x(z) - \sin\varphi_0 \, y(z)$$

$$y(\mathcal{R}_{\varphi_0} z) = \sin\varphi_0 \, x(z) + \cos\varphi_0 \, y(z)$$

generátory

$$\vec{X} = \frac{\vec{\partial}}{\partial x}$$

$$\vec{Y} = \frac{\vec{\partial}}{\partial y}$$

$$\vec{R} = -y\frac{\vec{\partial}}{\partial x} + x\frac{\vec{\partial}}{\partial y} = \frac{\vec{\partial}}{\partial \varphi}$$

algebra generátorů

$$[\vec{X}, \vec{R}] = \vec{Y} \qquad [\vec{Y}, \vec{R}] = -\vec{X} \qquad [\vec{X}, \vec{Y}] = 0$$

proof: $[\vec{X}, \vec{R}] = \left[\frac{\vec{\partial}}{\partial x}, -y\frac{\vec{\partial}}{\partial x} + x\frac{\vec{\partial}}{\partial y}\right] = \frac{\vec{\partial}}{\partial y} = \vec{Y}$

Lieova algebra $iso(E^2)$

obecně pro levou akci: $[\xi_M, \xi_N] = -c_{MN}{}^K \xi_K$

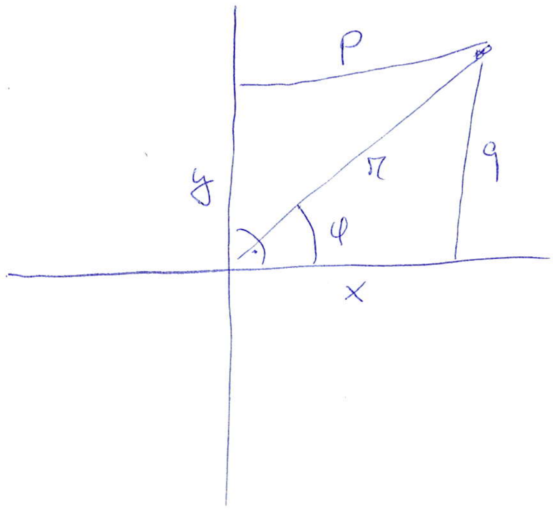
$\Rightarrow c_{XY}{}^R = 0 \quad c_{XR}{}^Y = -1 \quad c_{YR}{}^X = 1$ ← netriviální složky

$k_{RR} = -\frac{1}{2} c_{RX}{}^Y c_{RY}{}^X - \frac{1}{2} c_{RY}{}^X c_{RX}{}^Y = 1$ ostatní $= 0$

$$k_{\Gamma\Delta} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

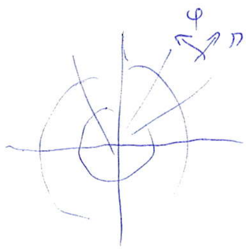
Maximálně symetrické prostory
Lobačevského rovina

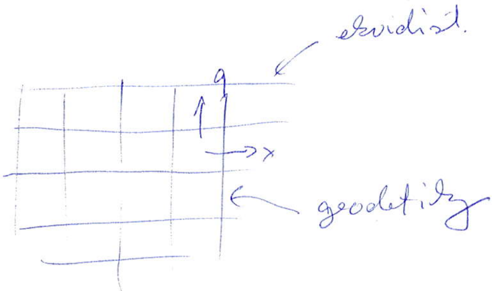
$$g = dr^2 + \mathrm{sh}^2 r\, d\varphi^2$$



$$\mathrm{ch}\, r = \mathrm{ch}\, x\, \mathrm{ch}\, q$$
$$\tan\varphi = \frac{\mathrm{th}\, q}{\mathrm{sh}\, x}$$

$$g = \mathrm{ch}^2 q\, dx + dq^2$$





$$\mathrm{th}\, x = \mathrm{th}\, r \cos\varphi$$
$$\mathrm{th}\, y = \mathrm{th}\, r \sin\varphi$$

$$\mathrm{sh}\, p = \mathrm{sh}\, r \cos\varphi$$
$$\mathrm{sh}\, q = \mathrm{sh}\, r \sin\varphi$$

$$\mathrm{th}^2 r = \mathrm{th}^2 x + \mathrm{th}^2 y$$
$$\mathrm{sh}^2 r = \mathrm{sh}^2 p + \mathrm{sh}^2 q$$

$$\mathrm{th}\, p = \mathrm{th}\, x\, \mathrm{ch}\, y$$
$$\mathrm{th}\, q = \mathrm{th}\, y\, \mathrm{ch}\, x$$

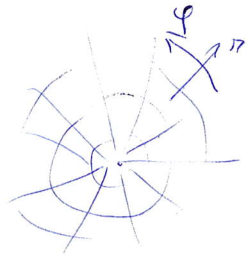
$$\frac{\mathrm{ch}\, x}{\mathrm{ch}\, y} = \frac{\mathrm{ch}\, p}{\mathrm{ch}\, q}$$

$$\mathrm{sh}\, x = \frac{\mathrm{sh}\, p}{\mathrm{ch}\, q}$$

$$\mathrm{sh}\, y = \frac{\mathrm{sh}\, q}{\mathrm{ch}\, p}$$

$$\frac{\mathrm{sh}\, p}{\mathrm{sh}\, q} = \frac{\mathrm{th}\, x}{\mathrm{th}\, y}$$

Isometrie Lobačevského roviny



$$g_{L^2} = dr^2 + \operatorname{sh}^2 r \, d\varphi^2$$

$$R = \frac{\partial}{\partial \varphi}$$

$$\mathcal{L}_R \, g_{L^2} = 0$$ R Killingův vektor

$$X = \cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\operatorname{th} r} \frac{\partial}{\partial \varphi}$$

$$\begin{aligned}
\mathcal{L}_X \, g_{L^2} &= \mathcal{L}_X \left(dr^2 + \operatorname{sh}^2 r \, d\varphi^2 \right) = \\
&= dr \vee \mathcal{L}_X dr + \left(\mathcal{L}_X \operatorname{sh}^2 r \right) d\varphi^2 + \operatorname{sh}^2 r \, d\varphi \vee \mathcal{L}_X d\varphi \\
&= dr \vee d(X[r]) + 2 \operatorname{sh} r \operatorname{ch} r \, X[r] \, d\varphi^2 + \operatorname{sh}^2 r \, d\varphi \vee d X[\varphi] \\
&= dr \vee d(\cos\varphi) + \operatorname{sh} 2r \cos\varphi \, d\varphi^2 - \operatorname{sh}^2 r \, d\varphi \vee d\left(\sin\varphi \frac{1}{\operatorname{th} r}\right) \\
&= -\sin\varphi \, dr \vee d\varphi + \operatorname{sh} 2r \cos\varphi \, d\varphi^2 + \sin\varphi \frac{\operatorname{sh}^2 r}{\operatorname{th}^2 r \operatorname{ch}^2 r} d\varphi \vee dr \\
&\quad - 2 \operatorname{sh} r \operatorname{ch} r \cos\varphi \, d\varphi^2 \\
&= 0
\end{aligned}$$

X je Killingův vektor

$$Y = \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\operatorname{th} r} \frac{\partial}{\partial \varphi}$$

$$\mathcal{L}_Y \, g_{L^2} = 0$$ obdobně X

Y je Killingův vektor

Lieova algebra symetrií

$$R = \frac{\partial}{\partial \varphi} = \operatorname{sh} x \frac{\partial}{\partial q} - \operatorname{ch} x \operatorname{th} q \frac{\partial}{\partial x}$$

$$X = \cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial \varphi} = \frac{\partial}{\partial x}$$

$$Y = \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial \varphi} = \operatorname{ch} x \frac{\partial}{\partial q} - \operatorname{sh} x \operatorname{th} q \frac{\partial}{\partial x}$$

$$[X, Y] = R$$

$$[X, R] = Y$$

$$[R, Y] = X$$

lineární komb. s konst. koef. Kill. v. je Kill. v.

$$V = V^R R + V^X X + V^Y Y \qquad V^R, V^X, V^Y \in \mathbb{R}$$

Lieova z. Kill. v. je Kill. v.

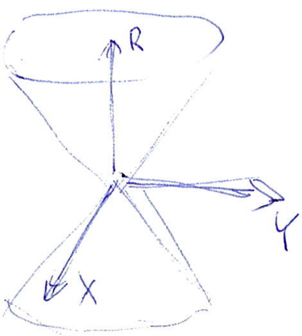
$$[\xi_\mu, \xi_\nu] = -C_{\mu\nu}{}^{\kappa}\, \xi_\kappa \qquad \xi_X = X \quad \xi_Y = Y \quad \xi_R = R$$

$$C_{XY}{}^{R} = -C_{YX}{}^{R} = -1 \qquad C_{XR}{}^{Y} = -C_{RX}{}^{Y} = -1 \qquad C_{RY}{}^{X} = -C_{YR}{}^{X} = -1$$

Killingova metrika

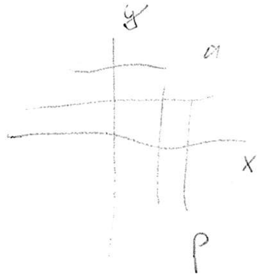
$$2K_{\alpha\beta} = -C_{\alpha\mu}{}^{\nu}\, C_{\beta\nu}{}^{\mu}$$

$$-K_{XX} = -K_{YY} = K_{RR} = 1 \qquad \operatorname{sign} K = (- - +)$$



LA alg. izometrií

$\xi \quad \upsilon \quad \varsigma$



$$X = \cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{th\, r}\frac{\partial}{\partial \varphi}$$

$$Y = + \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{th\, r}\frac{\partial}{\partial \varphi}$$

$$R = \frac{\partial}{\partial \varphi}$$

$\varphi = \frac{\pi}{2} \quad r = r_0$

$-\frac{1}{th\, r_0}\frac{\partial}{\partial\varphi}$

$$[X, Y] = \left[\cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{th\, r}\frac{\partial}{\partial \varphi},\ \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{th\, r}\frac{\partial}{\partial \varphi}\right]$$

$$= \left[\cos\varphi \frac{\partial}{\partial r},\ \cos\varphi \frac{1}{th\, r}\frac{\partial}{\partial \varphi}\right]$$

$$+ \left[-\sin\varphi \frac{1}{th\, r}\frac{\partial}{\partial \varphi},\ \sin\varphi \frac{\partial}{\partial r}\right]$$

$$\frac{1}{th^2 r}\left[-\sin\varphi \frac{\partial}{\partial\varphi},\ \cos\varphi \frac{\partial}{\partial\varphi}\right]$$

$$= \cos^2\varphi \frac{\partial\, th^{-1} r}{\partial r}\frac{\partial}{\partial\varphi} + \sin^2\varphi \frac{\partial\, th^{-1} r}{\partial r}\frac{\partial}{\partial\varphi}$$

$$+ \frac{1}{th^2 r}\sin^2\varphi \frac{\partial}{\partial\varphi} - \frac{\cos\varphi}{th^2 r}\left[\sin\varphi \frac{\partial}{\partial\varphi},\ \frac{\partial}{\partial\varphi}\right]$$

$$= -\frac{1}{th^2 r}\frac{1}{ch^2 r}\frac{\partial}{\partial\varphi} + \frac{1}{th^2 r}\frac{\partial}{\partial\varphi}$$

$$= \frac{1}{\cancel{th^2 r}}\left(\frac{\cancel{sh^2 r}}{\cancel{ch^2 r}}\right)\frac{\partial}{\partial\varphi} = R$$

$$C_{XY}{}^{R} = 1$$

$$C_{XR}{}^{Y} = 1$$

$$C_{RY}{}^{X} = 1$$

$$[X, Y] = R$$

$$[X, R] = \left[\cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{th\, r}\frac{\partial}{\partial\varphi},\ \frac{\partial}{\partial\varphi}\right]$$

$$= \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{th\, r}\frac{\partial}{\partial\varphi} = Y$$

$$[Y, R] = \left[\sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{th\, r}\frac{\partial}{\partial\varphi},\ \frac{\partial}{\partial\varphi}\right]$$

$$= -\left(\cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{th\, r}\frac{\partial}{\partial\varphi}\right] \overset{?}{=} -X$$

$$K_{\alpha\beta} = -\frac{1}{2} C_{\alpha\mu}{}^{\gamma} C_{\beta\gamma}{}^{\mu}$$

$$K_{XX} = -\frac{1}{2} 2 (C_{XY}{}^{R} C_{XR}{}^{Y} = -1$$

$$K_{YY} = -\frac{1}{2} 2 C_{YX}{}^{R} C_{YR}{}^{X} = -1$$

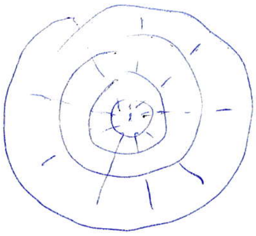
$$K_{RR} = -\frac{1}{2} 2 C_{RX}{}^{Y} C_{RY}{}^{X} = 1$$

$$K_{XY} = K_{XR} = K_{YR} = 0$$

$$\text{sign}\, K = - - +$$

Orbity isometrií = cykly

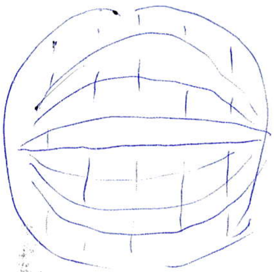
R



svazek skrze bod

cyklus = kružnice

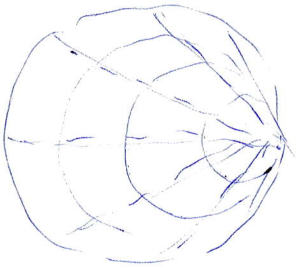
X, Y



svazek kolmý ve př.

cyklus = eqidistanta
= exocyklus

R ± X



svazek souběžek

cyklus = horocyklus

Akce isometrií na generátorech (KV)

komutační relace

$$[X,Y]=R \qquad [X,R]=Y \qquad [R,Y]=X$$

rotace generovaná R

$$[R,X]=-Y \qquad [R,Y]=X \quad \Rightarrow \quad C_R\cdot = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad C_R^2 = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\exp(\Delta\varphi\, C_R) = \cos\Delta\varphi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\Delta\varphi \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} V^X \\ V^Y \end{bmatrix} = \exp(\Delta\varphi\, C_R) \begin{bmatrix} V_0^X \\ V_0^Y \end{bmatrix}$$

$\Downarrow$

$$X_{\Delta\varphi} = R_{\Delta\varphi *} X = \cos\Delta\varphi\, X + \sin\Delta\varphi\, Y \qquad \Leftarrow V_0^X = 1 \quad V_0^Y = 0$$

$$Y_{\Delta\varphi} = R_{\Delta\varphi *} Y = -\sin\Delta\varphi\, X + \cos\Delta\varphi\, Y \qquad \Leftarrow V_0^X = 0 \quad V_0^Y = 1$$

boost generovaný X

$$[X,R]=Y \qquad [X,Y]=R \qquad C_X\cdot = -\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad C_X^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\exp(\Delta x\, C_X) = \mathrm{ch}\,\Delta x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \mathrm{sh}\,\Delta x \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} V^R \\ V^Y \end{bmatrix} = \exp(\Delta x\, C_X) \begin{bmatrix} V_0^R \\ V_0^Y \end{bmatrix}$$

$\Downarrow$

$$R_{\Delta x} = \mathcal{X}_{\Delta x *} R = \mathrm{ch}\,\Delta x\, R - \mathrm{sh}\,\Delta x\, Y \qquad \Leftarrow V_0^R = 1 \quad V_0^Y = 0$$

$$Y_{\Delta x} = \mathcal{X}_{\Delta x *} Y = -\mathrm{sh}\,\Delta x\, R + \mathrm{ch}\,\Delta x\, Y \qquad \Leftarrow V_0^R = 0 \quad V_0^Y = 1$$

boost generovaný Y

$$[Y,R]=-X \qquad [Y,X]=-R \qquad C_Y = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad C_Y^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\exp(\Delta y\, C_Y) = \mathrm{ch}\,\Delta y \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \mathrm{sh}\,\Delta y \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\Downarrow$

$$R_{\Delta y} = \mathcal{Y}_{\Delta y *} R = \mathrm{ch}\,\Delta y\, R + \mathrm{sh}\,\Delta y\, X \qquad \Leftarrow V_0^R = 1 \quad V_0^X = 0$$

$$X_{\Delta y} = \mathcal{Y}_{\Delta y *} X = \mathrm{sh}\,\Delta y\, R + \mathrm{ch}\,\Delta y\, X \qquad \Leftarrow V_0^R = 0 \quad V_0^X = 1$$

Grupa isometrií L^2

algebra generátorů isometrií na L^2

$$[X,Y] = R \qquad [R,X] = -Y \qquad [R,Y] = X$$

LA grupy isometrií $= iso(L^2)$

$$c_{XY}{}^{R} = 1 \qquad c_{RX}{}^{Y} = -1 \qquad c_{RY}{}^{X} = 1$$

$$-k_{XX} = -k_{YY} = k_{RR} = 1$$

akce na generátorech (tj. na LA)

$$X_{\Delta\varphi} = R_{\Delta\varphi *} X = \cos\Delta\varphi\, X + \sin\Delta\varphi\, Y$$
$$Y_{\Delta\varphi} = R_{\Delta\varphi *} Y = -\sin\Delta\varphi\, X + \cos\Delta\varphi\, Y$$

rotace v X-Y rovině vzhledem k metrice k

$$R_{\Delta x} = \mathcal{X}_{\Delta x *} R = \operatorname{ch}\Delta x\, R - \operatorname{sh}\Delta x\, Y$$
$$Y_{\Delta x} = \mathcal{X}_{\Delta x *} Y = -\operatorname{sh}\Delta x\, R + \operatorname{ch}\Delta x\, Y$$

boost v R-Y rovině vzhledem k metrice k

$$R_{\Delta y} = \mathcal{Y}_{\Delta y *} R = \operatorname{ch}\Delta y\, R + \operatorname{sh}\Delta y\, X$$
$$X_{\Delta y} = \mathcal{Y}_{\Delta y *} X = \operatorname{sh}\Delta y\, R + \operatorname{ch}\Delta y\, X$$

boost v R-X rovině vzhledem k metrice k

struktura grupy isometrií

isometrie L^2 $\Leftrightarrow$ isometrie Minkowského p. D=3

$R_{\Delta\varphi}, \mathcal{X}_{\Delta x}, \mathcal{Y}_{\Delta y}$ (prostor generátorů s metrikou k)

$\Downarrow$

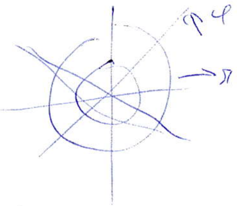
$$Iso(L^2) = SO(1,2)$$
$$iso(L^2) = so(1,2)$$

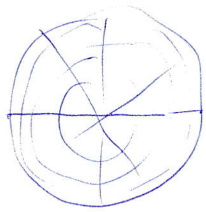
Přizpůsobené souřadnice

rotace R - polární souřadnice r, φ

$$g = dr^2 + \mathrm{sh}^2 r \, d\varphi^2$$

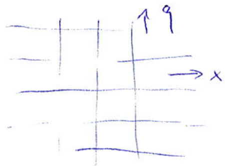
$$R = \frac{\partial}{\partial \varphi} \qquad r, \varphi$$

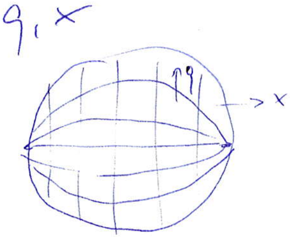




translace X - Lobač. souřadnice q, x

$$g = dq^2 + \mathrm{ch}^2 q \, dx^2$$





$$\mathrm{th}\, x = \mathrm{th}\, r \cos\varphi \qquad \mathrm{ch}\, r = \mathrm{ch}\, q \, \mathrm{ch}\, x \quad \text{(Plyt. v.)}$$

$$\mathrm{sh}\, q = \mathrm{sh}\, r \sin\varphi \qquad \tan\varphi = \frac{\mathrm{th}\, q}{\mathrm{sh}\, x}$$

$$\mathrm{sh}\, r \, dr = \mathrm{sh}\, q \, \mathrm{ch}\, x \, dq + \mathrm{ch}\, q \, \mathrm{sh}\, x \, dx$$

$$\mathrm{sh}^2 r \, d\varphi = \mathrm{sh}\, x \, dq - \mathrm{sh}\, q \, \mathrm{ch}\, q \, \mathrm{ch}\, x \, dx$$

kde $\mathrm{sh}^2 r = \mathrm{sh}^2 q + \mathrm{ch}^2 q \, \mathrm{sh}^2 x = \mathrm{sh}^2 q \, \mathrm{ch}^2 x + \mathrm{sh}^2 x = \mathrm{ch}^2 q \, \mathrm{ch}^2 x - 1$

$$\frac{1}{\mathrm{ch}\, x} \frac{\partial}{\partial q} = \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\mathrm{sh}\, r \, \mathrm{ch}\, r} \frac{\partial}{\partial \varphi}$$

$$\frac{\partial}{\partial x} = \cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\mathrm{th}\, r} \frac{\partial}{\partial \varphi} \qquad = X$$

$$\mathrm{th}\, r \frac{\partial}{\partial r} = \mathrm{th}\, q \frac{\partial}{\partial q} + \frac{\mathrm{th}\, x}{\mathrm{ch}^2 q} \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial \varphi} = \mathrm{sh}\, x \frac{\partial}{\partial q} - \mathrm{th}\, q \, \mathrm{ch}\, x \frac{\partial}{\partial x} \qquad = R$$

$$R = \mathrm{sh}\, x \frac{\partial}{\partial q} - \mathrm{ch}\, x \, \mathrm{th}\, q \frac{\partial}{\partial x}$$

$$X = \frac{\partial}{\partial x}$$

$$Y = \mathrm{ch}\, x \frac{\partial}{\partial q} - \mathrm{sh}\, x \, \mathrm{th}\, q \frac{\partial}{\partial x}$$

Isometrie

Rotace

$$R_{\Delta\varphi} : x \longrightarrow x_{\Delta\varphi} = R_{\Delta\varphi}\, x$$

$$r_{\Delta\varphi} = r \qquad\qquad \text{kde } r_{\Delta\varphi} = r(x_{\Delta\varphi}) \text{ atd.}$$

$$\varphi_{\Delta\varphi} = \varphi + \Delta\varphi \qquad\qquad r = r(x)$$

$$\frac{D x_{\Delta\varphi}}{d\Delta\varphi} = \frac{d r_{\Delta\varphi}}{d\Delta\varphi}\frac{\partial}{\partial r} + \frac{d\varphi_{\Delta\varphi}}{d\Delta\varphi}\frac{\partial}{\partial\varphi} = \frac{\partial}{\partial\varphi} = R$$

$$DR_{\Delta\varphi}\big|_x = \frac{\partial X^a_{\Delta\varphi}}{\partial x^b}\Big|_x dx^b\big|_x \frac{\partial}{\partial X^a}\Big|_{x_{\Delta\varphi}} = dr\big|_x \frac{\partial}{\partial r}\Big|_{x_{\Delta\varphi}} + d\varphi\big|_x \frac{\partial}{\partial\varphi}\Big|_{x_{\Delta\varphi}}$$

rotace KV X

$$X_{\Delta\varphi}\big|_{x_{\Delta\varphi}} = R_{\Delta\varphi *}\left(X\big|_x\right) = X\big|_x \cdot DR_{\Delta\varphi}\big|_x =$$

$$= \cos\varphi \frac{\partial}{\partial r}\Big|_{x_{\Delta\varphi}} - \sin\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi}\Big|_{x_{\Delta\varphi}}$$

platí

$$\frac{\partial}{\partial r} = \cos\varphi\, X + \sin\varphi\, Y$$

$$\frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi} = -\sin\varphi\, X + \cos\varphi\, X$$

$\Rightarrow$

$$X_{\Delta\varphi}\big|_{x_{\Delta\varphi}} = \cos\varphi\cos(\varphi+\Delta\varphi)\, X\big|_{x_{\Delta\varphi}} + \cos\varphi\sin(\varphi+\Delta\varphi)\, Y\big|_{x_{\Delta\varphi}}$$

$$+ \sin\varphi\sin(\varphi+\Delta\varphi)\, X\big|_{x_{\Delta\varphi}} - \sin\varphi\cos(\varphi+\Delta\varphi)\, Y\big|_{x_{\Delta\varphi}}$$

$$= \cos\Delta\varphi\, X\big|_{x_{\Delta\varphi}} + \sin\Delta\varphi\, Y\big|_{x_{\Delta\varphi}}$$

$$X_{\Delta\varphi} = \cos\Delta\varphi\, X + \sin\Delta\varphi\, Y$$

obdobně

$$Y_{\Delta\varphi} = -\sin\Delta\varphi\, X + \cos\Delta\varphi\, Y$$

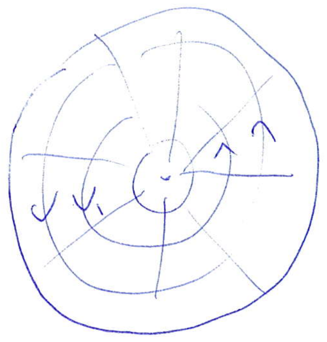
rotace KV R

$$R_{\Delta\varphi}\big|_{x_{\Delta\varphi}} = R_{\Delta\varphi *}\, R\big|_x =$$

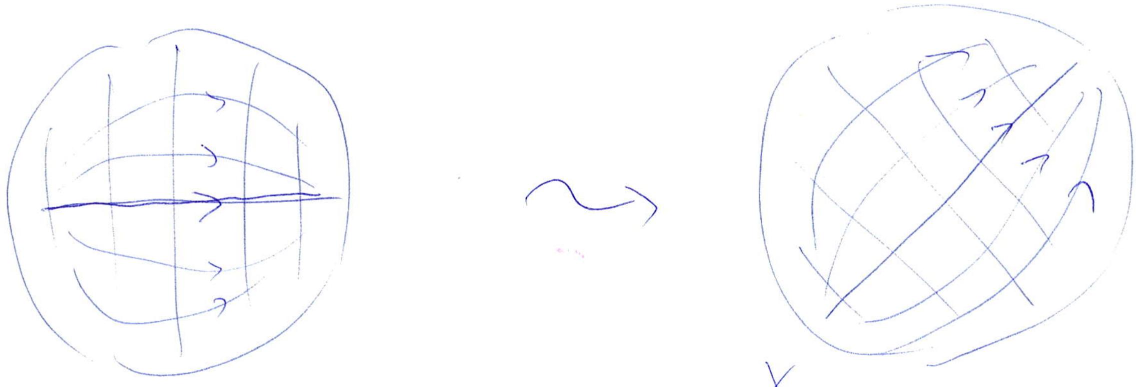
$$= \frac{\partial}{\partial\varphi}\Big|_{x_{\Delta\varphi}} = R\big|_{x_{\Delta\varphi}}$$

$\Downarrow$

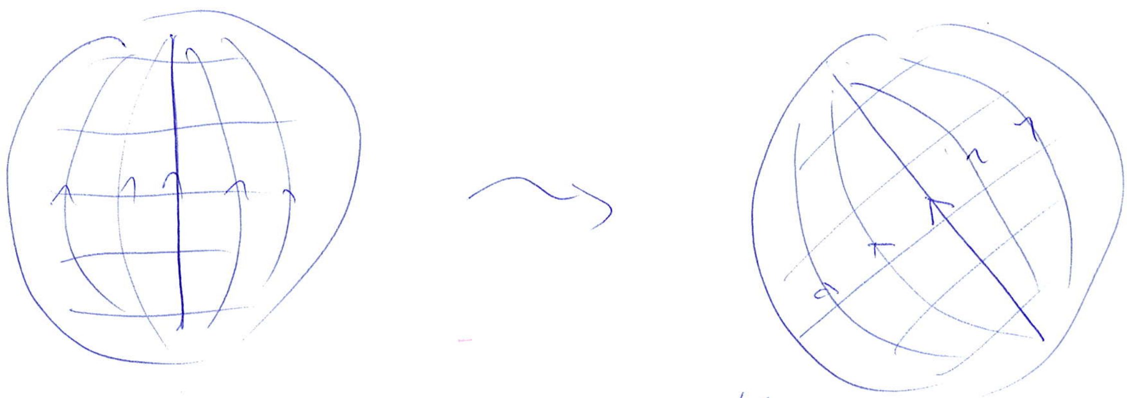
$$R_{\Delta\varphi} = R$$



R



X $\qquad\qquad$ $X_{\Delta\varphi}$



Y $\qquad\qquad$ $Y_{\Delta\varphi}$

Isometrie

translace ve směru X

$\mathcal{X}_{\Delta X} : x \longrightarrow x_{\Delta X} = \mathcal{X}_{\Delta X}\, x$

$q_{\Delta X} = q$ kde $q_{\Delta X} = q(x_{\Delta X})$ atd.

$X_{\Delta X} = X + \Delta X$ $\quad q = q(x)$

$$\frac{Dx_{\Delta X}}{d\Delta X} = \frac{dq_{\Delta X}}{d\Delta X}\frac{\partial}{\partial q} + \frac{\partial X_{\Delta X}}{\partial \Delta X}\frac{\partial}{\partial X} = \frac{\partial}{\partial X} = X$$

$$D\mathcal{X}_{\Delta X}|_x = \dots = dq_x \frac{\partial}{\partial q}\Big|_{x_{\Delta X}} + dX|_x \frac{\partial}{\partial X}\Big|_{x_{\Delta X}}$$

translace KV R

$$R_{\Delta X}|_{x_{\Delta X}} = \mathcal{X}_{\Delta X *}(R|_x) = R|_x \cdot D\mathcal{X}_{\Delta X}|_x =$$

$$= \operatorname{sh} X \frac{\partial}{\partial q}\Big|_{x_{\Delta X}} - \operatorname{ch} X \operatorname{thq} \frac{\partial}{\partial X}\Big|_{x_{\Delta X}}$$

platí

$$-\operatorname{thq}\frac{\partial}{\partial X} = \operatorname{ch} X\, R - \operatorname{sh} X\, Y$$

$$\frac{\partial}{\partial q} = -\operatorname{sh} X\, R + \operatorname{ch} X\, Y$$

$$R_{\Delta X}|_{x_{\Delta X}} = \Big(-\operatorname{sh} X \operatorname{sh}(X+\Delta X)\, R + \operatorname{sh} X \operatorname{ch}(X+\Delta X)\, Y + \operatorname{ch} X \operatorname{ch}(X+\Delta X)\, R - \operatorname{ch} X \operatorname{sh}(X+\Delta X)\, Y\Big)\Big|_{x_{\Delta X}}$$

$$= \operatorname{ch}\Delta X\, R|_{x_{\Delta X}} - \operatorname{sh}\Delta X\, Y|_{x_{\Delta X}}$$

$$R_{\Delta X} = \operatorname{ch}\Delta X\, R - \operatorname{sh}\Delta X\, Y$$

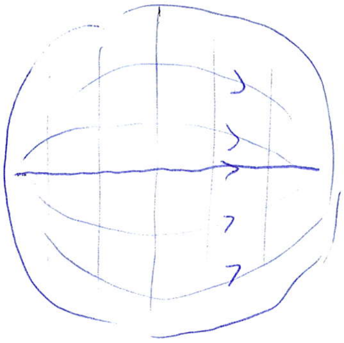
obdobně translace KV Y

$$Y_{\Delta X} = -\operatorname{sh}\Delta X\, R + \operatorname{ch}\Delta X\, Y$$

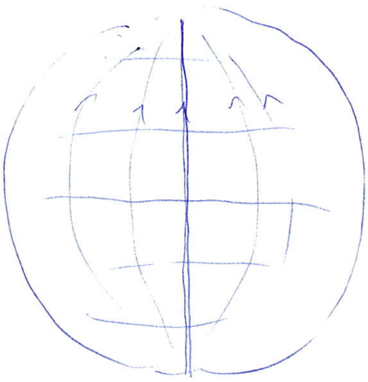
translace KV X

$$X_{\Delta X}|_{x_{\Delta X}} = \mathcal{X}_{\Delta X *}\, X|_x = \frac{\partial}{\partial X}\Big|_{x_{\Delta X}} = X|_{x_{\Delta X}}$$

$$X_{\Delta X} = X$$

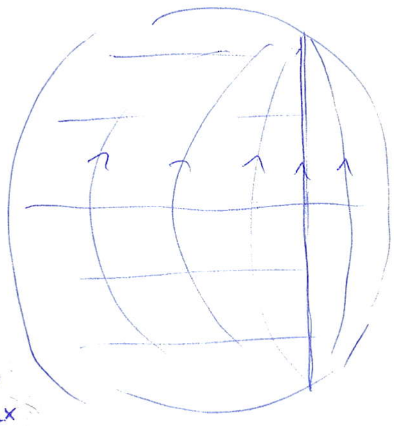


X

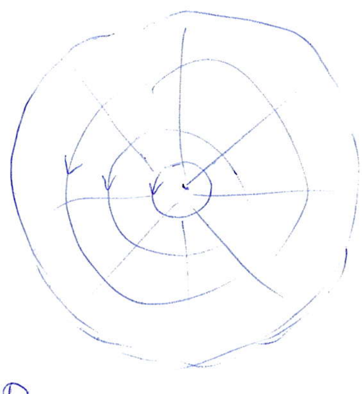


Y

$\rightsquigarrow$

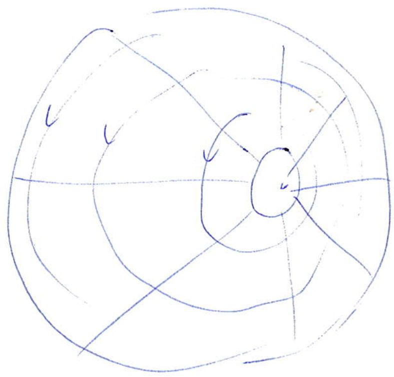


$Y_{\Delta x}$



R

$\rightsquigarrow$



$R_{\Delta x}$